

Package ‘sgs’

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Title Sparse-Group SLOPE: Adaptive Bi-Level Selection with FDR Control

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Description Implementation of Sparse-group SLOPE (SGS) (Feser and Evangelou (2023) <[doi:10.48550/arXiv.2305.09467](https://doi.org/10.48550/arXiv.2305.09467)>) models. Linear and logistic regression models are supported, both of which can be fit using k-fold cross-validation. Dense and sparse input matrices are supported. In addition, a general adaptive three operator splitting (ATOS) implementation is provided. Group SLOPE (gSLOPE) (Brzyski et al. (2019) <[doi:10.1080/01621459.2017.1411269](https://doi.org/10.1080/01621459.2017.1411269)>) models are also implemented. Both gSLOPE and SGS are available with strong screening rules (Feser and Evangelou (2024) <[doi:10.48550/arXiv.2405.15357](https://doi.org/10.48550/arXiv.2405.15357)>) for computational speed-up.

Imports Matrix, MASS, caret, grDevices, graphics, methods, stats, faux, SLOPE, Rlab, Rcpp (>= 1.0.10)

LinkingTo Rcpp, RcppArmadillo

Suggests SGL, gglasso, glmnet, testthat, knitr, grpSLOPE, rmarkdown

RoxygenNote 7.3.1

License GPL (>= 3)

Encoding UTF-8

URL <https://github.com/ff1201/sgs>

BugReports <https://github.com/ff1201/sgs/issues>

VignetteBuilder knitr

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arma_mv

Matrix Product in RcppArmadillo.

Description

Matrix Product in RcppArmadillo.

Usage

arma_mv(m, v)

Arguments

m	numeric matrix
v	numeric vector

Value

matrix product of m and v

arma_sparse	<i>Matrix Product in RcppArmadillo.</i>
-------------	---

Description

Matrix Product in RcppArmadillo.

Usage

```
arma_sparse(m, v)
```

Arguments

m	numeric sparse matrix
v	numeric vector

Value

matrix product of m and v

as_sgs	<i>Fits the adaptively scaled SGS model (AS-SGS).</i>
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Description

Fits an SGS model using the noise estimation procedure, termed adaptively scaled SGS (Algorithm 2 from Feser and Evangelou (2023)). This adaptively estimates λ and then fits the model using the estimated value. It is an alternative approach to cross-validation ([fit_sgs_cv\(\)](#)). The approach is only compatible with the SGS penalties.

Usage

```
as_sgs(  
  X,  
  y,  
  groups,  
  type = "linear",  
  pen_method = 2,  
  alpha = 0.95,  
  vFDR = 0.1,  
  gFDR = 0.1,  
  standardise = "l2",  
  intercept = TRUE,  
  verbose = FALSE  
)
```

Arguments

<code>X</code>	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
<code>y</code>	Output vector of dimension n . For <code>type="linear"</code> should be continuous and for <code>type="logistic"</code> should be a binary variable.
<code>groups</code>	A grouping structure for the input data. Should take the form of a vector of group indices.
<code>type</code>	The type of regression to perform. Supported values are: "linear" and "logistic".
<code>pen_method</code>	The type of penalty sequences to use. <ul style="list-style-type: none"> • "1" uses the vMean and gMean SGS sequences. • "2" uses the vMax and gMax SGS sequences.
<code>alpha</code>	The value of α , which defines the convex balance between SLOPE and gSLOPE. Must be between 0 and 1.
<code>vFDR</code>	Defines the desired variable false discovery rate (FDR) level, which determines the shape of the variable penalties. Must be between 0 and 1.
<code>gFDR</code>	Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties. Must be between 0 and 1.
<code>standardise</code>	Type of standardisation to perform on X : <ul style="list-style-type: none"> • "l2" standardises the input data to have ℓ_2 norms of one. • "l1" standardises the input data to have ℓ_1 norms of one. • "sd" standardises the input data to have standard deviation of one. • "none" no standardisation applied.
<code>intercept</code>	Logical flag for whether to fit an intercept.
<code>verbose</code>	Logical flag for whether to print fitting information.

Value

An object of type "sgs" containing model fit information (see `fit_sgs()`).

References

Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, <https://arxiv.org/abs/2305.09467>

See Also

`scaled_sgs()`

Other model-selection: `fit_gslope_cv()`, `fit_sgs_cv()`, `scaled_sgs()`

Other SGS-methods: `coef.sgs()`, `fit_sgs()`, `fit_sgs_cv()`, `plot.sgs()`, `predict.sgs()`, `print.sgs()`, `scaled_sgs()`

 atos

Adaptive three operator splitting (ATOS).

Description

Function for fitting adaptive three operator splitting (ATOS) with general convex penalties. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
atos(
  X,
  y,
  type = "linear",
  prox_1,
  prox_2,
  pen_prox_1 = 0.5,
  pen_prox_2 = 0.5,
  max_iter = 5000,
  backtracking = 0.7,
  max_iter_backtracking = 100,
  tol = 1e-05,
  prox_1_opts = NULL,
  prox_2_opts = NULL,
  standardise = "l2",
  intercept = TRUE,
  x0 = NULL,
  u = NULL,
  verbose = FALSE
)
```

Arguments

X	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package)
y	Output vector of dimension n . For type="linear" needs to be continuous and for type="logistic" needs to be a binary variable.
type	The type of regression to perform. Supported values are: "linear" and "logistic".
prox_1	The proximal operator for the first function, $h(x)$.
prox_2	The proximal operator for the second function, $g(x)$.
pen_prox_1	The penalty for the first proximal operator. For the lasso, this would be the sparsity parameter, λ . If operator does not include a penalty, set to 1.
pen_prox_2	The penalty for the second proximal operator.
max_iter	Maximum number of ATOS iterations to perform.
backtracking	The backtracking parameter, τ , as defined in Pedregosa et. al. (2018).

max_iter_backtracking	Maximum number of backtracking line search iterations to perform per global iteration.
tol	Convergence tolerance for the stopping criteria.
prox_1_opts	Optional argument for first proximal operator. For the group lasso, this would be the group IDs. Note: this must be inserted as a list.
prox_2_opts	Optional argument for second proximal operator.
standardise	Type of standardisation to perform on X : <ul style="list-style-type: none"> • "l2" standardises the input data to have ℓ_2 norms of one. • "l1" standardises the input data to have ℓ_1 norms of one. • "sd" standardises the input data to have standard deviation of one. • "none" no standardisation applied.
intercept	Logical flag for whether to fit an intercept.
x0	Optional initial vector for x_0 .
u	Optional initial vector for u .
verbose	Logical flag for whether to print fitting information.

Details

atos() solves convex minimization problems of the form

$$f(x) + g(x) + h(x),$$

where f is convex and differentiable with L_f -Lipschitz gradient, and g and h are both convex. The algorithm is not symmetrical, but usually the difference between variations are only small numerical values, which are filtered out. However, both variations should be checked regardless, by looking at x and u . An example for the sparse-group lasso (SGL) is given.

Value

An object of class "atos" containing:

beta	The fitted values from the regression. Taken to be the more stable fit between x and u , which is usually the former.
x	The solution to the original problem (see Pedregosa et. al. (2018)).
u	The solution to the dual problem (see Pedregosa et. al. (2018)).
z	The updated values from applying the first proximal operator (see Pedregosa et. al. (2018)).
type	Indicates which type of regression was performed.
success	Logical flag indicating whether ATOS converged, according to tol.
num_it	Number of iterations performed. If convergence is not reached, this will be max_iter.
certificate	Final value of convergence criteria.
intercept	Logical flag indicating whether an intercept was fit.

References

Pedregosa, F., Gidel, G. (2018). *Adaptive Three Operator Splitting*, <https://proceedings.mlr.press/v80/pedregosa18a.html>

coef.sgs	<i>Extracts coefficients for one of the following object types: "sgs", "sgs_cv", "gslope", "gslope_cv".</i>
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Description

Print the coefficients using model fitted with one of the following functions: `fit_sgs()`, `fit_sgs_cv()`, `fit_gslope()`, `fit_gslope_cv()`. The predictions are calculated for each "lambda" value in the path.

Usage

```
## S3 method for class 'sgs'
coef(object, ...)
```

Arguments

`object` Object of one of the following classes: "sgs", "sgs_cv", "gslope", "gslope_cv".
`...` further arguments passed to stats function.

Value

The fitted coefficients

See Also

`fit_sgs()`, `fit_sgs_cv()`, `fit_gslope()`, `fit_gslope_cv()`

Other SGS-methods: `as_sgs()`, `fit_sgs()`, `fit_sgs_cv()`, `plot.sgs()`, `predict.sgs()`, `print.sgs()`, `scaled_sgs()`

Other gSLOPE-methods: `fit_gslope()`, `fit_gslope_cv()`, `plot.sgs()`, `predict.sgs()`, `print.sgs()`

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGS
model = fit_sgs(X = data$X, y = data$y, groups = groups, type="linear", lambda = 1, alpha=0.95,
vFDR=0.1, gFDR=0.1, standardise = "l2", intercept = TRUE, verbose=FALSE)
# use predict function
model_coef = coef(model)
```

fit_gslope

Fit a gSLOPE model.

Description

Group SLOPE (gSLOPE) main fitting function. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_gslope(
  X,
  y,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
  min_frac = 0.05,
  gFDR = 0.1,
  pen_method = 1,
  max_iter = 5000,
  backtracking = 0.7,
  max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "l2",
  intercept = TRUE,
  screen = TRUE,
  verbose = FALSE,
  w_weights = NULL
)
```

Arguments

X	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
y	Output vector of dimension n . For type="linear" should be continuous and for type="logistic" should be a binary variable.
groups	A grouping structure for the input data. Should take the form of a vector of group indices.
type	The type of regression to perform. Supported values are: "linear" and "logistic".
lambda	The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models: <ul style="list-style-type: none"> "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac".

	<ul style="list-style-type: none"> • User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).
path_length	The number of λ values to fit the model for. If "lambda" is user-specified, this is ignored.
min_frac	Defines the termination point of the pathwise solution, so that $\lambda_{\min} = \text{min_frac} \cdot \lambda_{\max}$.
gFDR	Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties. Must be between 0 and 1.
pen_method	The type of penalty sequences to use (see Brzyski et al. (2019)): <ul style="list-style-type: none"> • "1" uses the gMean gSLOPE sequence. • "2" uses the gMax gSLOPE sequence.
max_iter	Maximum number of ATOS iterations to perform.
backtracking	The backtracking parameter, τ , as defined in Pedregosa et. al. (2018).
max_iter_backtracking	Maximum number of backtracking line search iterations to perform per global iteration.
tol	Convergence tolerance for the stopping criteria.
standardise	Type of standardisation to perform on X : <ul style="list-style-type: none"> • "l2" standardises the input data to have ℓ_2 norms of one. When using this "lambda" is scaled internally by $1/\sqrt{n}$. • "l1" standardises the input data to have ℓ_1 norms of one. When using this "lambda" is scaled internally by $1/n$. • "sd" standardises the input data to have standard deviation of one. • "none" no standardisation applied.
intercept	Logical flag for whether to fit an intercept.
screen	Logical flag for whether to apply screening rules (see Feser and Evangelou (2024)). Screening discards irrelevant groups before fitting, greatly improving speed.
verbose	Logical flag for whether to print fitting information.
w_weights	Optional vector for the group penalty weights. Overrides the penalties from pen_method if specified. When entering custom weights, these are multiplied internally by λ and $1 - \alpha$. To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$.

Details

`fit_gslope()` fits a gSLOPE model using adaptive three operator splitting (ATOS). gSLOPE is a sparse-group method, so that it selects both variables and groups. Unlike group selection approaches, not every variable within a group is set as active. It solves the convex optimisation problem given by

$$\frac{1}{2n} f(b; y, \mathbf{X}) + \lambda \sum_{g=1}^m w_g \sqrt{p_g} \|b^{(g)}\|_2,$$

where the penalty sequences are sorted and $f(\cdot)$ is the loss function. In the case of the linear model, the loss function is given by the mean-squared error loss:

$$f(b; y, \mathbf{X}) = \|y - \mathbf{X}b\|_2^2.$$

In the logistic model, the loss function is given by

$$f(b; y, \mathbf{X}) = -1/n \log(\mathcal{L}(b; y, \mathbf{X})).$$

where the log-likelihood is given by

$$\mathcal{L}(b; y, \mathbf{X}) = \sum_{i=1}^n \{y_i b^\top x_i - \log(1 + \exp(b^\top x_i))\}.$$

The penalty parameters in gSLOPE are sorted so that the largest group effects are matched with the largest penalties, to reduce the group FDR.

Value

A list containing:

beta	The fitted values from the regression. Taken to be the more stable fit between x and z, which is usually the former. A filter is applied to remove very small values, where ATOS has not been able to shrink exactly to zero. Check this against x and z.
group_effects	The group values from the regression. Taken by applying the ℓ_2 norm within each group on beta.
selected_var	A list containing the indicies of the active/selected variables for each "lambda" value.
selected_grp	A list containing the indicies of the active/selected groups for each "lambda" value.
pen_gslope	Vector of the group penalty sequence.
lambda	Value(s) of λ used to fit the model.
type	Indicates which type of regression was performed.
standardise	Type of standardisation used.
intercept	Logical flag indicating whether an intercept was fit.
num_it	Number of iterations performed. If convergence is not reached, this will be max_iter.
success	Logical flag indicating whether ATOS converged, according to tol.
certificate	Final value of convergence criteria.
x	The solution to the original problem (see Pedregosa et. al. (2018)).
u	The solution to the dual problem (see Pedregosa et. al. (2018)).
z	The updated values from applying the first proximal operator (see Pedregosa et. al. (2018)).
screen_set	List of groups that were kept after screening step for each "lambda" value. (corresponds to \mathcal{S} in Feser and Evangelou (2024)).

epsilon_set	List of groups that were used for fitting after screening for each "lambda" value. (corresponds to \mathcal{E} in Feser and Evangelou (2024)).
kkt_violations	List of groups that violated the KKT conditions each "lambda" value. (corresponds to \mathcal{K} in Feser and Evangelou (2024)).
screen	Logical flag indicating whether screening was applied.

References

- Brzyski, D., Gossmann, A., Su, W., Bodgan, M. (2019). *Group SLOPE – Adaptive Selection of Groups of Predictors*, <https://www.tandfonline.com/doi/full/10.1080/01621459.2017.1411269>
- Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, <https://proceedings.mlr.press/v80/pedregosa18a.html>
- Pedregosa, F., Gidel, G. (2018). *Adaptive Three Operator Splitting*, <https://proceedings.mlr.press/v80/pedregosa18a.html>

See Also

Other gSLOPE-methods: `coef.sgs()`, `fit_gslope_cv()`, `plot.sgs()`, `predict.sgs()`, `print.sgs()`

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run gSLOPE
model = fit_gslope(X = data$X, y = data$y, groups = groups, type="linear", path_length = 5,
gFDR=0.1, standardise = "l2", intercept = TRUE, verbose=FALSE)
```

fit_gslope_cv

Fit a gSLOPE model using k-fold cross-validation.

Description

Function to fit a pathwise solution of group SLOPE (gSLOPE) models using k-fold cross-validation. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_gslope_cv(
  X,
  y,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
```

```

min_frac = 0.05,
nfolds = 10,
gFDR = 0.1,
pen_method = 1,
backtracking = 0.7,
max_iter = 5000,
max_iter_backtracking = 100,
tol = 1e-05,
standardise = "l2",
intercept = TRUE,
error_criteria = "mse",
screen = TRUE,
verbose = FALSE,
w_weights = NULL
)

```

Arguments

X	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
y	Output vector of dimension n . For type="linear" should be continuous and for type="logistic" should be a binary variable.
groups	A grouping structure for the input data. Should take the form of a vector of group indices.
type	The type of regression to perform. Supported values are: "linear" and "logistic".
lambda	The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models: <ul style="list-style-type: none"> "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac". User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).
path_length	The number of λ values to fit the model for. If "lambda" is user-specified, this is ignored.
min_frac	Defines the termination point of the pathwise solution, so that $\lambda_{\min} = \text{min_frac} \cdot \lambda_{\max}$.
nfolds	The number of folds to use in cross-validation.
gFDR	Defines the desired group false discovery rate (FDR) level, which determines the shape of the penalties. Must be between 0 and 1.
pen_method	The type of penalty sequences to use (see Brzyski et al. (2019)): <ul style="list-style-type: none"> "1" uses the gMean gSLOPE sequence. "2" uses the gMax gSLOPE sequence.
backtracking	The backtracking parameter, τ , as defined in Pedregosa et. al. (2018).
max_iter	Maximum number of ATOS iterations to perform.

max_iter_backtracking	Maximum number of backtracking line search iterations to perform per global iteration.
tol	Convergence tolerance for the stopping criteria.
standardise	Type of standardisation to perform on X : <ul style="list-style-type: none"> • "l2" standardises the input data to have ℓ_2 norms of one. • "l1" standardises the input data to have ℓ_1 norms of one. • "sd" standardises the input data to have standard deviation of one. • "none" no standardisation applied.
intercept	Logical flag for whether to fit an intercept.
error_criteria	The criteria used to discriminate between models along the path. Supported values are: "mse" (mean squared error) and "mae" (mean absolute error).
screen	Logical flag for whether to apply screening rules (see Feser and Evangelou (2024)). Screening discards irrelevant groups before fitting, greatly improving speed.
verbose	Logical flag for whether to print fitting information.
w_weights	Optional vector for the group penalty weights. Overrides the penalties from pen_method if specified. When entering custom weights, these are multiplied internally by λ . To void this behaviour, set $\lambda = 1$ or scale it accordingly.

Details

Fits gSLOPE models under a pathwise solution using adaptive three operator splitting (ATOS), picking the 1se model as optimum. Warm starts are implemented.

Value

A list containing:

errors	A table containing fitting information about the models on the path.
all_models	Fitting information for all models fit on the path, which is a "gslope" object type.
fit	The 1se chosen model, which is a "gslope" object type.
best_lambda	The value of λ which generated the chosen model.
best_lambda_id	The path index for the chosen model.

References

- Brzyski, D., Gossmann, A., Su, W., Bodgan, M. (2019). *Group SLOPE – Adaptive Selection of Groups of Predictors*, <https://www.tandfonline.com/doi/full/10.1080/01621459.2017.1411269>
- Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, <https://proceedings.mlr.press/v80/pedregosa18a.html>

See Also

[fit_gslope\(\)](#)

Other gSLOPE-methods: [coef.sgs\(\)](#), [fit_gslope\(\)](#), [plot.sgs\(\)](#), [predict.sgs\(\)](#), [print.sgs\(\)](#)

Other model-selection: [as_sgs\(\)](#), [fit_sgs_cv\(\)](#), [scaled_sgs\(\)](#)

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run gSLOPE with cross-validation (the proximal functions can be found in utils.R)
cv_model = fit_gslope_cv(X = data$X, y = data$y, groups=groups, type = "linear", path_length = 5,
nolds=5, gFDR = 0.1, min_frac = 0.05, standardise="l2",intercept=TRUE,verbose=TRUE)
```

fit_sgs

Fit an SGS model.

Description

Sparse-group SLOPE (SGS) main fitting function. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_sgs(
  X,
  y,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
  min_frac = 0.05,
  alpha = 0.95,
  vFDR = 0.1,
  gFDR = 0.1,
  pen_method = 1,
  max_iter = 5000,
  backtracking = 0.7,
  max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "l2",
  intercept = TRUE,
  screen = TRUE,
  verbose = FALSE,
  w_weights = NULL,
  v_weights = NULL
)
```

Arguments

X	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
y	Output vector of dimension n . For type="linear" should be continuous and for type="logistic" should be a binary variable.
groups	A grouping structure for the input data. Should take the form of a vector of group indices.
type	The type of regression to perform. Supported values are: "linear" and "logistic".
lambda	The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models: <ul style="list-style-type: none"> "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac". User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).
path_length	The number of λ values to fit the model for. If "lambda" is user-specified, this is ignored.
min_frac	Smallest value of λ as a fraction of the maximum value. That is, the final λ will be "min_frac" of the first λ value.
alpha	The value of α , which defines the convex balance between SLOPE and gSLOPE. Must be between 0 and 1. Recommended value is 0.95.
vFDR	Defines the desired variable false discovery rate (FDR) level, which determines the shape of the variable penalties. Must be between 0 and 1.
gFDR	Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties. Must be between 0 and 1.
pen_method	The type of penalty sequences to use (see Feser and Evangelou (2023)): <ul style="list-style-type: none"> "1" uses the vMean SGS and gMean gSLOPE sequences. "2" uses the vMax SGS and gMean gSLOPE sequences. "3" uses the BH SLOPE and gMean gSLOPE sequences, also known as SGS Original.
max_iter	Maximum number of ATOS iterations to perform.
backtracking	The backtracking parameter, τ , as defined in Pedregosa et. al. (2018).
max_iter_backtracking	Maximum number of backtracking line search iterations to perform per global iteration.
tol	Convergence tolerance for the stopping criteria.
standardise	Type of standardisation to perform on X: <ul style="list-style-type: none"> "l2" standardises the input data to have ℓ_2 norms of one. When using this "lambda" is scaled internally by $1/\sqrt{n}$. "l1" standardises the input data to have ℓ_1 norms of one. When using this "lambda" is scaled internally by $1/n$.

	<ul style="list-style-type: none"> • "sd" standardises the input data to have standard deviation of one. • "none" no standardisation applied.
intercept	Logical flag for whether to fit an intercept.
screen	Logical flag for whether to apply screening rules (see Feser and Evangelou (2024)). Screening discards irrelevant groups before fitting, greatly improving speed.
verbose	Logical flag for whether to print fitting information.
w_weights	Optional vector for the group penalty weights. Overrides the penalties from pen_method if specified. When entering custom weights, these are multiplied internally by λ and $1 - \alpha$. To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$.
v_weights	Optional vector for the variable penalty weights. Overrides the penalties from pen_method if specified. When entering custom weights, these are multiplied internally by λ and α . To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$.

Details

`fit_sgs()` fits an SGS model using adaptive three operator splitting (ATOS). SGS is a sparse-group method, so that it selects both variables and groups. Unlike group selection approaches, not every variable within a group is set as active. It solves the convex optimisation problem given by

$$\frac{1}{2n} f(b; y, \mathbf{X}) + \lambda \alpha \sum_{i=1}^p v_i |b|_{(i)} + \lambda (1 - \alpha) \sum_{g=1}^m w_g \sqrt{p_g} \|b^{(g)}\|_2,$$

where the penalty sequences are sorted and $f(\cdot)$ is the loss function. In the case of the linear model, the loss function is given by the mean-squared error loss:

$$f(b; y, \mathbf{X}) = \|y - \mathbf{X}b\|_2^2.$$

In the logistic model, the loss function is given by

$$f(b; y, \mathbf{X}) = -1/n \log(\mathcal{L}(b; y, \mathbf{X})).$$

where the log-likelihood is given by

$$\mathcal{L}(b; y, \mathbf{X}) = \sum_{i=1}^n \{y_i b^\top x_i - \log(1 + \exp(b^\top x_i))\}.$$

SGS can be seen to be a convex combination of SLOPE and gSLOPE, balanced through alpha, such that it reduces to SLOPE for alpha = 0 and to gSLOPE for alpha = 1. The penalty parameters in SGS are sorted so that the largest coefficients are matched with the largest penalties, to reduce the FDR.

Value

A list containing:

beta	The fitted values from the regression. Taken to be the more stable fit between x and z, which is usually the former. A filter is applied to remove very small values, where ATOS has not been able to shrink exactly to zero. Check this against x and z.
------	---

x	The solution to the original problem (see Pedregosa et. al. (2018)).
u	The solution to the dual problem (see Pedregosa et. al. (2018)).
z	The updated values from applying the first proximal operator (see Pedregosa et. al. (2018)).
type	Indicates which type of regression was performed.
pen_slope	Vector of the variable penalty sequence.
pen_gslope	Vector of the group penalty sequence.
lambda	Value(s) of λ used to fit the model.
success	Logical flag indicating whether ATOS converged, according to tol.
num_it	Number of iterations performed. If convergence is not reached, this will be max_iter.
certificate	Final value of convergence criteria.
intercept	Logical flag indicating whether an intercept was fit.

References

- Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, <https://arxiv.org/abs/2305.09467>
- Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, <https://arxiv.org/abs/2405.15357>
- Pedregosa, F., Gidel, G. (2018). *Adaptive Three Operator Splitting*, <https://proceedings.mlr.press/v80/pedregosa18a.html>

See Also

Other SGS-methods: [as_sgs\(\)](#), [coef.sgs\(\)](#), [fit_sgs_cv\(\)](#), [plot.sgs\(\)](#), [predict.sgs\(\)](#), [print.sgs\(\)](#), [scaled_sgs\(\)](#)

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGS
model = fit_sgs(X = data$X, y = data$y, groups = groups, type="linear", path_length = 5, alpha=0.95,
vFDR=0.1, gFDR=0.1, standardise = "l2", intercept = TRUE, verbose=FALSE)
```

fit_sgs_cv

Fit an SGS model using k-fold cross-validation.

Description

Function to fit a pathwise solution of sparse-group SLOPE (SGS) models using k-fold cross-validation. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_sgs_cv(
  X,
  y,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
  min_frac = 0.05,
  alpha = 0.95,
  vFDR = 0.1,
  gFDR = 0.1,
  pen_method = 1,
  nfolds = 10,
  backtracking = 0.7,
  max_iter = 5000,
  max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "l2",
  intercept = TRUE,
  error_criteria = "mse",
  screen = TRUE,
  verbose = FALSE,
  v_weights = NULL,
  w_weights = NULL
)
```

Arguments

X	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
y	Output vector of dimension n . For type="linear" should be continuous and for type="logistic" should be a binary variable.
groups	A grouping structure for the input data. Should take the form of a vector of group indices.
type	The type of regression to perform. Supported values are: "linear" and "logistic".

lambda	<p>The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models:</p> <ul style="list-style-type: none"> • "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac". • User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).
path_length	The number of λ values to fit the model for. If "lambda" is user-specified, this is ignored.
min_frac	Defines the termination point of the pathwise solution, so that $\lambda_{\min} = \text{min_frac} \cdot \lambda_{\max}$.
alpha	The value of α , which defines the convex balance between SLOPE and gSLOPE. Must be between 0 and 1. Recommended value is 0.95.
vFDR	Defines the desired variable false discovery rate (FDR) level, which determines the shape of the variable penalties. Must be between 0 and 1.
gFDR	Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties. Must be between 0 and 1.
pen_method	<p>The type of penalty sequences to use (see Feser and Evangelou (2023)):</p> <ul style="list-style-type: none"> • "1" uses the vMean SGS and gMean gSLOPE sequences. • "2" uses the vMax SGS and gMean gSLOPE sequences. • "3" uses the BH SLOPE and gMean gSLOPE sequences, also known as SGS Original.
nfolds	The number of folds to use in cross-validation.
backtracking	The backtracking parameter, τ , as defined in Pedregosa et. al. (2018).
max_iter	Maximum number of ATOS iterations to perform.
max_iter_backtracking	Maximum number of backtracking line search iterations to perform per global iteration.
tol	Convergence tolerance for the stopping criteria.
standardise	<p>Type of standardisation to perform on X:</p> <ul style="list-style-type: none"> • "l2" standardises the input data to have ℓ_2 norms of one. • "l1" standardises the input data to have ℓ_1 norms of one. • "sd" standardises the input data to have standard deviation of one. • "none" no standardisation applied.
intercept	Logical flag for whether to fit an intercept.
error_criteria	The criteria used to discriminate between models along the path. Supported values are: "mse" (mean squared error) and "mae" (mean absolute error).
screen	Logical flag for whether to apply screening rules (see Feser and Evangelou (2024)). Screening discards irrelevant groups before fitting, greatly improving speed.
verbose	Logical flag for whether to print fitting information.

v_weights	Optional vector for the variable penalty weights. Overrides the penalties from pen_method if specified. When entering custom weights, these are multiplied internally by λ and α . To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$
w_weights	Optional vector for the group penalty weights. Overrides the penalties from pen_method if specified. When entering custom weights, these are multiplied internally by λ and $1 - \alpha$. To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$

Details

Fits SGS models under a pathwise solution using adaptive three operator splitting (ATOS), picking the 1se model as optimum. Warm starts are implemented.

Value

A list containing:

all_models	A list of all the models fitted along the path.
fit	The 1se chosen model, which is a "sgs" object type.
best_lambda	The value of λ which generated the chosen model.
best_lambda_id	The path index for the chosen model.
errors	A table containing fitting information about the models on the path.
type	Indicates which type of regression was performed.

References

Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, <https://arxiv.org/abs/2305.09467>

Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, <https://arxiv.org/abs/2405.15357>

See Also

[fit_sgs\(\)](#)

Other model-selection: [as_sgs\(\)](#), [fit_gslope_cv\(\)](#), [scaled_sgs\(\)](#)

Other SGS-methods: [as_sgs\(\)](#), [coef.sgs\(\)](#), [fit_sgs\(\)](#), [plot.sgs\(\)](#), [predict.sgs\(\)](#), [print.sgs\(\)](#), [scaled_sgs\(\)](#)

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGS with cross-validation (the proximal functions can be found in utils.R)
cv_model = fit_sgs_cv(X = data$X, y = data$y, groups=groups, type = "linear",
path_length = 5, nolds=5, alpha = 0.95, vFDR = 0.1, gFDR = 0.1, min_frac = 0.05,
standardise="l2", intercept=TRUE, verbose=TRUE)
```

gen_pens	<i>Generate penalty sequences for SGS.</i>
----------	--

Description

Generates variable and group penalties for SGS.

Usage

```
gen_pens(gFDR, vFDR, pen_method, groups, alpha)
```

Arguments

gFDR	Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties.
vFDR	Defines the desired variable false discovery rate (FDR) level, which determines the shape of the variable penalties.
pen_method	The type of penalty sequences to use (see Feser and Evangelou (2023)): <ul style="list-style-type: none"> • "1" uses the vMean SGS and gMean gSLOPE sequences. • "2" uses the vMax SGS and gMean gSLOPE sequences. • "3" uses the BH SLOPE and gMean gSLOPE sequences, also known as SGS Original. • "4" uses the gMax gSLOPE sequence. For a gSLOPE model only.
groups	A grouping structure for the input data. Should take the form of a vector of group indices.
alpha	The value of α , defines the convex balance between SLOPE and gSLOPE.

Details

The vMean and vMax SGS sequences are variable sequences derived specifically to give variable false discovery rate (FDR) control for SGS under orthogonal designs (see Feser and Evangelou (2023)). The BH SLOPE sequence is derived in Bodgan et. al. (2015) and has links to the Benjamini-Hochberg critical values. The sequence provides variable FDR-control for SLOPE under orthogonal designs. The gMean gSLOPE sequence is derived in Brzyski et. al. (2015) and provides group FDR-control for gSLOPE under orthogonal designs.

Value

A list containing:

pen_slope_org	A vector of the variable penalty sequence.
pen_gslope_org	A vector of the group penalty sequence.

References

- Bogdan, M., Van den Berg, E., Sabatti, C., Su, W., Candes, E. (2015). *SLOPE — Adaptive variable selection via convex optimization*, <https://projecteuclid.org/journals/annals-of-applied-statistics/volume-9/issue-3/SLOPEAdaptive-variable-selection-via-convex-optimization/10.1214/15-AOAS842.full>
- Brzyski, D., Gossmann, A., Su, W., Bodgan, M. (2019). *Group SLOPE – Adaptive Selection of Groups of Predictors*, <https://www.tandfonline.com/doi/full/10.1080/01621459.2017.1411269>
- Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, <https://arxiv.org/abs/2305.09467>

Examples

```
# specify a grouping structure
groups = c(rep(1:20, each=3),
           rep(21:40, each=4),
           rep(41:60, each=5),
           rep(61:80, each=6),
           rep(81:100, each=7))
# generate sequences
sequences = gen_pens(gFDR=0.1, vFDR=0.1, pen_method=1, groups=groups, alpha=0.5)
```

gen_toy_data

Generate toy data.

Description

Generates different types of datasets, which can then be fitted using sparse-group SLOPE.

Usage

```
gen_toy_data(
  p,
  n,
  rho = 0,
  seed_id = 2,
  grouped = TRUE,
  groups,
  noise_level = 1,
  group_sparsity = 0.1,
  var_sparsity = 0.5,
  orthogonal = FALSE,
  data_mean = 0,
  data_sd = 1,
  signal_mean = 0,
  signal_sd = sqrt(10)
)
```

Arguments

p	The number of input variables.
n	The number of observations.
rho	Correlation coefficient. Must be in range $[0, 1]$.
seed_id	Seed to be used to generate the data matrix X .
grouped	A logical flag indicating whether grouped data is required.
groups	If grouped=TRUE, the grouping structure is required. Each input variable should have a group id.
noise_level	Defines the level of noise (<i>sigma</i>) to be used in generating the response vector y .
group_sparsity	Defines the level of group sparsity. Must be in the range $[0, 1]$.
var_sparsity	Defines the level of variable sparsity. Must be in the range $[0, 1]$. If grouped=TRUE, this defines the level of sparsity within each group, not globally.
orthogonal	Logical flag as to whether the input matrix should be orthogonal.
data_mean	Defines the mean of input predictors.
data_sd	Defines the standard deviation of the signal (<i>beta</i>).
signal_mean	Defines the mean of the signal (<i>beta</i>).
signal_sd	Defines the standard deviation of the signal (<i>beta</i>).

Details

The data is generated under a Gaussian linear model. The generated data can be grouped and sparsity can be provided at both a group and/or variable level.

Value

A list containing:

y	The response vector.
X	The input matrix.
true_beta	The true values of <i>beta</i> used to generate the response.
true_grp_id	Indices of which groups are non-zero in true_beta.

Examples

```
# specify a grouping structure
groups = c(rep(1:20, each=3),
           rep(21:40, each=4),
           rep(41:60, each=5),
           rep(61:80, each=6),
           rep(81:100, each=7))

# generate data
data = gen_toy_data(p=500, n=400, groups = groups, seed_id=3)
```

plot.sgs	<i>Plot models of the following object types: "sgs", "sgs_cv", "gslope", "gslope_cv".</i>
----------	---

Description

Plots the pathwise solution of a cross-validation fit, from a call to one of the following: [fit_sgs\(\)](#), [fit_sgs_cv\(\)](#), [fit_gslope\(\)](#), [fit_gslope_cv\(\)](#).

Usage

```
## S3 method for class 'sgs'
plot(x, how_many = 10, ...)
```

Arguments

x	Object of one of the following classes: "sgs", "sgs_cv", "gslope", "gslope_cv".
how_many	Defines how many predictors to plot. Plots the predictors in decreasing order of largest absolute value.
...	further arguments passed to base function.

Value

A list containing:

response	The predicted response. In the logistic case, this represents the predicted class probabilities.
class	The predicted class assignments. Only returned if type = "logistic" in the model object.

See Also

[fit_sgs\(\)](#), [fit_sgs_cv\(\)](#), [fit_gslope\(\)](#), [fit_gslope_cv\(\)](#)

Other SGS-methods: [as_sgs\(\)](#), [coef.sgs\(\)](#), [fit_sgs\(\)](#), [fit_sgs_cv\(\)](#), [predict.sgs\(\)](#), [print.sgs\(\)](#), [scaled_sgs\(\)](#)

Other gSLOPE-methods: [coef.sgs\(\)](#), [fit_gslope\(\)](#), [fit_gslope_cv\(\)](#), [predict.sgs\(\)](#), [print.sgs\(\)](#)

Examples

```
# specify a grouping structure
groups = c(1,1,2,2,3)
# generate data
data = gen_toy_data(p=5, n=4, groups = groups, seed_id=3, signal_mean=20, group_sparsity=1)
# run SGS
model = fit_sgs(X = data$X, y = data$y, groups=groups, type = "linear",
path_length = 20, alpha = 0.95, vFDR = 0.1, gFDR = 0.1,
min_frac = 0.05, standardise="l2", intercept=TRUE, verbose=FALSE)
plot(model, how_many = 10)
```

predict.sgs	<i>Predict using one of the following object types: "sgs", "sgs_cv", "gslope", "gslope_cv".</i>
-------------	---

Description

Performs prediction from one of the following fits: [fit_sgs\(\)](#), [fit_sgs_cv\(\)](#), [fit_gslope\(\)](#), [fit_gslope_cv\(\)](#). The predictions are calculated for each "lambda" value in the path.

Usage

```
## S3 method for class 'sgs'
predict(object, x, ...)
```

Arguments

object	Object of one of the following classes: "sgs", "sgs_cv", "gslope", "gslope_cv".
x	Input data to use for prediction.
...	further arguments passed to stats function.

Value

A list containing:

response	The predicted response. In the logistic case, this represents the predicted class probabilities.
class	The predicted class assignments. Only returned if type = "logistic" in the "sgs" object.

See Also

[fit_sgs\(\)](#), [fit_sgs_cv\(\)](#), [fit_gslope\(\)](#), [fit_gslope_cv\(\)](#)

Other SGS-methods: [as_sgs\(\)](#), [coef.sgs\(\)](#), [fit_sgs\(\)](#), [fit_sgs_cv\(\)](#), [plot.sgs\(\)](#), [print.sgs\(\)](#), [scaled_sgs\(\)](#)

Other gSLOPE-methods: [coef.sgs\(\)](#), [fit_gslope\(\)](#), [fit_gslope_cv\(\)](#), [plot.sgs\(\)](#), [print.sgs\(\)](#)

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGS
model = fit_sgs(X = data$X, y = data$y, groups = groups, type="linear", lambda = 1, alpha=0.95,
vFDR=0.1, gFDR=0.1, standardise = "l2", intercept = TRUE, verbose=FALSE)
# use predict function
model_predictions = predict(model, x = data$X)
```

print.sgs	<i>Print a "sgs" object.</i>
-----------	------------------------------

Description

Performs prediction from an `fit_sgs()` model fit.

Usage

```
## S3 method for class 'sgs'  
print(x, ...)
```

Arguments

`x` Object an object of class "sgs" from a call to `fit_sgs()` or `fit_sgs_cv()`.
`...` further arguments passed to base function.

Value

A summary of the model fit(s).

See Also

`fit_sgs()`, `fit_sgs_cv()`, `fit_gslope()`, `fit_gslope_cv()`

Other SGS-methods: `as_sgs()`, `coef.sgs()`, `fit_sgs()`, `fit_sgs_cv()`, `plot.sgs()`, `predict.sgs()`, `scaled_sgs()`

Other gSLOPE-methods: `coef.sgs()`, `fit_gslope()`, `fit_gslope_cv()`, `plot.sgs()`, `predict.sgs()`

Examples

```
# specify a grouping structure  
groups = c(rep(1:20, each=3),  
           rep(21:40, each=4),  
           rep(41:60, each=5),  
           rep(61:80, each=6),  
           rep(81:100, each=7))  
  
# generate data  
data = gen_toy_data(p=500, n=400, groups = groups, seed_id=3)  
  
# run SGS  
model = fit_sgs(X = data$X, y = data$y, groups = groups, type="linear", lambda = 1, alpha=0.95,  
               vFDR=0.1, gFDR=0.1, standardise = "l2", intercept = TRUE, verbose=FALSE)  
  
# print model  
print(model)
```

scaled_sgs	<i>Fits a scaled SGS model.</i>
------------	---------------------------------

Description

Fits an SGS model using the noise estimation procedure (Algorithm 5 from Bogdan et. al. (2015)). This estimates λ and then fits the model using the estimated value. It is an alternative approach to cross-validation (`fit_sgs_cv()`).

Usage

```
scaled_sgs(
  X,
  y,
  groups,
  type = "linear",
  pen_method = 1,
  alpha = 0.95,
  vFDR = 0.1,
  gFDR = 0.1,
  standardise = "l2",
  intercept = TRUE,
  verbose = FALSE
)
```

Arguments

X	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
y	Output vector of dimension n . For type="linear" should be continuous and for type="logistic" should be a binary variable.
groups	A grouping structure for the input data. Should take the form of a vector of group indices.
type	The type of regression to perform. Supported values are: "linear" and "logistic".
pen_method	The type of penalty sequences to use. <ul style="list-style-type: none"> "1" uses the vMean SGS and gMean gSLOPE sequences. "2" uses the vMax SGS and gMean gSLOPE sequences. "1" uses the BH SLOPE and gMean gSLOPE sequences, also known as SGS Original.
alpha	The value of α , which defines the convex balance between SLOPE and gSLOPE. Must be between 0 and 1.
vFDR	Defines the desired variable false discovery rate (FDR) level, which determines the shape of the variable penalties. Must be between 0 and 1.
gFDR	Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties. Must be between 0 and 1.

standardise	Type of standardisation to perform on X : <ul style="list-style-type: none"> • "l2" standardises the input data to have ℓ_2 norms of one. • "l1" standardises the input data to have ℓ_1 norms of one. • "sd" standardises the input data to have standard deviation of one. • "none" no standardisation applied.
intercept	Logical flag for whether to fit an intercept.
verbose	Logical flag for whether to print fitting information.

Value

An object of type "sgs" containing model fit information (see [fit_sgs\(\)](#)).

References

Bogdan, M., Van den Berg, E., Sabatti, C., Su, W., Candes, E. (2015). *SLOPE — Adaptive variable selection via convex optimization*, <https://projecteuclid.org/journals/annals-of-applied-statistics/volume-9/issue-3/SLOPEAdaptive-variable-selection-via-convex-optimization/10.1214/15-A0AS842.full>

See Also

[as_sgs\(\)](#)

Other model-selection: [as_sgs\(\)](#), [fit_gslope_cv\(\)](#), [fit_sgs_cv\(\)](#)

Other SGS-methods: [as_sgs\(\)](#), [coef.sgs\(\)](#), [fit_sgs\(\)](#), [fit_sgs_cv\(\)](#), [plot.sgs\(\)](#), [predict.sgs\(\)](#), [print.sgs\(\)](#)

Examples

```
# specify a grouping structure
groups = c(1,1,2,2,3)
# generate data
data = gen_toy_data(p=5, n=4, groups = groups, seed_id=3,
  signal_mean=20,group_sparsity=1,var_sparsity=1)
# run noise estimation
model = scaled_sgs(X=data$X, y=data$y, groups=groups, pen_method=1)
```

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