

# Package ‘steadyICA’

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**Type** Package

**Title** ICA and Tests of Independence via Multivariate Distance  
Covariance

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**Description** Functions related to multivariate measures of independence and ICA:  
-estimate independent components by minimizing distance covariance;  
-conduct a test of mutual independence based on distance covariance;  
-estimate independent components via infomax (a popular method but generally performs poorer than mdcovica, ProDenICA, and/or fastICA, but is useful for comparisons);  
-order independent components by skewness;  
-match independent components from multiple estimates;  
-other functions useful in ICA.

**License** GPL (>= 2)

**Depends** Rcpp (>= 0.9.13), MASS, clue, combinat

**Suggests** irlba, JADE, ProDenICA, fastICA, energy

**LinkingTo** Rcpp

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## R topics documented:

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|                   |   |
|-------------------|---|
| steadyICA-package | <i>ICA via distance covariance, tests of mutual independence, and other ICA functions</i> |
|-------------------|---|

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## Description

Functions related to multivariate measures of independence and ICA:

- estimate independent components by minimizing distance covariance;
- conduct a test of mutual independence based on distance covariance;
- estimate independent components via infomax (a popular method but generally performs poorer than steadyICA or ProDenICA but is useful for comparisons);
- order independent components by skewness;
- match independent components from multiple estimates;
- other functions useful in ICA.

## Details

|           |                                 |
|-----------|---------------------------------|
| Package:  | steadyICA                       |
| Type:     | Package                         |
| Version:  | 1.0                             |
| Date:     | 2015-11-08                      |
| License:  | GPL (>= 2)                      |
| Depends:  | Rcpp (>= 0.9.13), MASS          |
| Suggests: | irlba, JADE, ProDenICA, fastICA |

## Author(s)

Benjamin B. Risk and Nicholas A. James and David S. Matteson.  
 Maintainer: Benjamin Risk <bbr28@cornell.edu>

## References

Bernaards, C. & Jennrich, R. (2005) Gradient projection algorithms and software for arbitrary rotation criteria in factor analysis. *Educational and Psychological Measurement* 65, 676-696

Matteson, D. S. & Tsay, R. Independent component analysis via U-Statistics. <<http://www.stat.cornell.edu/~matteson/#ICA>>  
 Szekely, G., Rizzo, M. & Bakirov, N. Measuring and testing dependence by correlation of distances. (2007) *The Annals of Statistics*, 35, 2769-2794.  
 Tichavsky, P. & Koldovsky, Z. Optimal pairing of signal components separated by blind techniques. (2004) *Signal Processing Letters* 11, 119-122.

### See Also

[fastICA](#) [ProDenICA](#): :ProDenICA

### Examples

#see steadyICA

---

compInd

*Complete Measure of Mutual Multivariate Independence*

---

### Description

Calculates a complete empirical measure of mutual multivariate independence. Makes use of the `utils::combn` function.

### Usage

```
compInd(S, group=1:ncol(S), alpha=1)
```

### Arguments

|                    |  |
|--------------------|--|
| <code>S</code>     | The $n \times d$ matrix for which you wish to calculate the dependence between $d$ columns from $n$ samples. |
| <code>group</code> | A length $d$ vector which indicates group membership for each component.                                     |
| <code>alpha</code> | The index used in calculating the distance between sample observations.                                      |

### Value

Returns a scalar equal to the empirical multivariate distance between the observed samples, and their grouped counterpart.

### Note

Suppose that the each component belongs to exactly one of  $C$  groups. This method makes use of the `utils::combn` and `combinat::permn` functions. As a result it will be both computationally and memory intensive, even for small to moderate  $n$  and small  $C$ .

### Author(s)

Nicholas James

**References**

Chasalow, Scott (2012) combinat: Combinatorics Utilities <<http://CRAN.R-project.org/package=combinat>

**See Also**

[dcovustat](#), [energy::dcov](#)

**Examples**

```
library(steadyICA)
library(combinat)
set.seed(100)
S = matrix(rnorm(40), ncol=4)
group = c(1, 2, 3, 3)
compInd(S, group, 1)
```

---

 dcovICA

---

*ICA via distance covariance for 2 components*


---

**Description**

This algorithm finds the rotation which minimizes the distance covariance between two orthogonal components via the angular parameterization of a 2x2 orthogonal matrix with the function `stats::optimize`. The results will be (approximately) equivalent to `steadyICA` but this function is much faster (but does not extend to higher dimensions).

**Usage**

```
dcovICA(Z, theta.0 = 0)
```

**Arguments**

|         |   |
|---------|---|
| Z       | The whitened $n \times d$ data matrix, where $n$ is the number of observations and $d$ the number of components.  |
| theta.0 | Determines the interval to be searched by the optimizer: lower bound = <code>theta.0</code> , upper bound = $\pi/2$ . Changing <code>theta.0</code> affects the initial value, where the initial value = $\text{theta.0} + (1/2 + \sqrt{5}/2) * \pi/2$ , see <a href="#">optimize</a> . |

**Value**

|           |   |
|-----------|---|
| theta.hat | Estimated minimum.                        |
| W         | $W = t(\text{theta}2W(\text{theta.hat}))$ |
| S         | Estimated independent components.         |
| obj       | The distance covariance of S.             |

**Author(s)**

David Matteson and Benjamin Risk

**References**

Matteson, D. S. & Tsay, R. Independent component analysis via U-Statistics. <<http://www.stat.cornell.edu/~matteson/#ICA>>

**See Also**

[steadyICA](#), [optimize](#)

**Examples**

```
library(JADE)
library(ProDenICA)
set.seed(123)
simS = cbind(rjordan(letter='j',n=1024),rjordan(letter='m',n=1024))
simM = mixmat(p=2)
xData = simS%%simM
xWhitened = whitener(xData)

#Define true unmixing matrix as true M multiplied by the estimated whitener:
#Call this the target matrix:
W.true <- solve(simM%%xWhitened$whitener)

a=Sys.time()
est.dCovICA = dcovICA(Z = xWhitened$Z,theta.0=0)
Sys.time()-a

#See the example with steadyICA for an explanation
#of the parameterization used in amari.error:
amari.error(t(est.dCovICA$W),W.true)

##NOTE: also try theta.0 = pi/4 since there may be local minima
## Not run: est.dcovICA = dcovICA(Z = xWhitened$Z,theta.0=pi/4)
## amari.error(t(est.dcovICA$W),W.true)
## End(Not run)

a=Sys.time()
est.steadyICA = steadyICA(X=xWhitened$Z,verbose=TRUE)
Sys.time()-a
amari.error(t(est.steadyICA$W),W.true)
##theta parameterization with optimize is much faster
```

**Description**

Calculates the square of the U-statistic formulation of distance covariance. This is faster than the function 'dcov' in the R package 'energy' and requires less memory. Note that negative values are possible in this version.

**Usage**

```
dcovustat(x,y,alpha=1)
```

**Arguments**

|       |  |
|-------|--|
| x     | A vector or matrix.  |
| y     | A vector or matrix with the same number of observations as x, though the number of columns of x and y may differ |
| alpha | A scaling parameter in the interval (0,2] used for calculating distances.  |

**Value**

Returns the distance covariance U-statistic.

**Note**

The value returned by dcovustat is equal to the square of the value returned by energy::dcov in the limit.

In dcovustat, a vector of length n is stored; in energy::dcov, an n x n matrix is stored. Thus, dcovustat requires far less memory and works for very large datasets.

Even though dcovustat converges to the square of the distance covariance of the random variables x and y, it can be negative.

**Author(s)**

David Matteson

**References**

Matteson, D. S. & Tsay, R. Independent component analysis via U-Statistics. <<http://www.stat.cornell.edu/~matteson/#ICA>>  
Szekely, G., Rizzo, M. & Bakirov, N. Measuring and testing dependence by correlation of distances. (2007) *The Annals of Statistics*, 35, 2769-2794.

**See Also**

[multidcov](#), [energy::dcov](#)

**Examples**

```

x = rnorm(5000)
y = rbinom(5000,1,0.5)
y = y - 1*(y==0)
z = y*exp(-x) #some non-linear dependence

dcovustat(x[1:1000],y[1:1000]) #close to zero

a = Sys.time()
dcovustat(x[1:1000],z[1:1000]) #greater than zero
a = Sys.time() - a

#measures of linear dependence close to zero:
cov(x,z)
cor(rank(x),rank(z))

## Not run:
#dcovustat differs from energy::dcov but are equal in the limit
library(energy)
b = Sys.time()
(dcov(x[1:1000],z[1:1000]))^2
b = Sys.time() - b
as.double(b)/as.double(a) #dcovustat is much faster

## energy::dcov and dcovustat become approximately equal as n increases:
c = Sys.time()
dcovustat(x,z)
c = difftime(Sys.time(), c, sec)
d = Sys.time()
(dcov(x,z))^2
d = difftime(Sys.time(), d, sec)
as.double(d)/as.double(c)

## End(Not run)

```

---

frobICA

*match mixing matrices or ICs and calculate their Frobenius distance*


---

**Description**

The ICA model is only identifiable up to signed permutations of the ICs. This function provides a similarity measure between two mixing matrices for the model  $X = S M + E$ , where  $X$  is  $n \times p$ ,  $S$  is  $n \times d$ , and  $M$  is  $d \times p$ . The input is either two mixing matrices  $M1$  and  $M2$  or two matrices of independent components  $S1$  and  $S2$ . For  $M1$  and  $M2$ , `frobICA()` finds the signed row permutation of  $M2$  that minimizes the Frobenius norm between  $M1$  and  $M2$  using the Hungarian method. For  $S1$  and  $S2$ , `frobICA()` finds the signed column permutation of  $S2$  that minimizes the Frobenius norm between  $S1$  and  $S2$ . This function allows the mixing matrices (or independent components) to have differing numbers of rows (respectively, columns) such that the similarity measure is defined by the matching rows (resp., columns), and the non-matching rows (resp., columns) are discarded.

**Usage**

```
frobICA(M1 = NULL, M2 = NULL, S1 = NULL, S2 = NULL, standardize = FALSE)
```

**Arguments**

|             |   |
|-------------|---|
| M1          | A d x p mixing matrix                     |
| M2          | A d x q mixing matrix                     |
| S1          | An n x d matrix of independent components |
| S2          | An n x q matrix of independent components |
| standardize | Logical. See Note.                        |

**Details**

$\text{frobICA}(M1, M2) = 0$  if there exists a signed permutation of the rows of  $M2$  such that  $M1 = P \% \% M2$ , where  $P$  is a  $d \times q$  signed permutation matrix, i.e., composed of 0, 1, and -1, with  $d \leq q$ ; the function also allows  $d > q$ , in which case  $\text{frobICA}(M1, M2) = 0$  if there exists a  $P$  such that  $P \% \% M1 = M2$ . Unlike other ICA performance measures, this function can accommodate non-square mixing matrices.

**Value**

returns the Frobenius norm divided by  $p \cdot \min(d, q)$  (or  $n \cdot \min(d, q)$ ) of the matched mixing matrices (resp., matched independent components).

**Note**

If `standardize=TRUE`, then scales the rows of  $M1$  and  $M2$  to have unit norm or the columns of  $S1$  and  $S2$  to have zero mean and sample variance equal to one. The user can supply either  $M1$  and  $M2$  or  $S1$  and  $S2$  but not both.

**Author(s)**

Benjamin Risk

**References**

Kuhn, H. The Hungarian Method for the assignment problem *Naval Research Logistics Quarterly*, 1955, 2, 83 - 97

Risk, B.B., D.S. Matteson, D. Ruppert, A. Eloyan, B.S. Caffo. In review, 2013. Evaluating ICA methods with an application to resting state fMRI.

**See Also**

[JADE::MD clue::solve\\_LSAP matchICA](#)



**Examples**

```

mat1 <- matrix(rnorm(4*6),nrow=4)
perm <- matrix(c(-1,0,0,0,0,0,1,0,0,1,0,0,0,0,1),4,4)
mat2 <- perm%*%mat1
sqrt(sum((mat1-mat2)^2))
frobICA(M1=mat1,M2=mat2)

#Another example showing invariance to permutations:
covMat <- t(mat1)%*%mat1
mvsample <- matrix(rnorm(400),100,4)%*%mat1
frobICA(M1=cov(mvsample),M2=covMat)
frobICA(M1=cov(mvsample),M2=covMat[sample(1:6),])

#Example using independent components:
nObs=300
simS<-cbind(rgamma(nObs, shape = 1, scale = 2),
            gamma(nObs, shape = 3, scale = 2),
            gamma(nObs, shape = 3, scale = 2),
            gamma(nObs, shape = 9, scale = 0.5))

#not necessary in this example, but this should be done when used with ICA:
simS <- apply(simS,2,scale)
frobICA(S1=simS,S2=simS%*%perm)
## Not run:
#returns an error if S1 and S2 are not explicitly defined:
frobICA(simS,simS%*%perm)

## End(Not run)

```

gmultidcov

*Symmetric multivariate distance covariance for grouped components***Description**

Calculate either the symmetric or asymmetric multivariate distance covariance statistic for a given grouping of the components.

**Usage**

```
gmultidcov(S,group=1:ncol(S),alpha=1,symmetric=TRUE)
```

**Arguments**

|       |  |
|-------|--|
| S     | The n x d matrix for which you wish to calculate the dependence between d columns from n samples |
| group | A length d vector which indicates group membership for each component                            |

**alpha**            A scaling parameter in the interval (0,2] used for calculating distances.  
**symmetric**       logical; if TRUE (the default), calculates the symmetric version of the multivariate distance covariance. See details.

### Details

Suppose that the groups are numbered 1,2,...,C and that group is a vector indicating group membership for each component. If symmetric==TRUE, calculates:  $\sum_{i=1}^C \text{dcovustat}(S[, \text{group}==i], S[, \text{group}!=i])$   
 If symmetric==FALSE, calculates:  $\sum_{i=1}^{C-1} \text{dcovustat}(S[, \text{group}==i], S[, \text{group}>i])$

### Value

Returns a scalar equal to the multivariate distance covariance statistic for grouped components of S.

### Author(s)

Nicholas James

### See Also

[dcovustat](#), [energy::dcov](#)

### Examples

```
library(steadyICA)
S = matrix(rnorm(300), ncol=3)
group = c(1,2,2)
gmultidcov(S,group,TRUE) # close to zero
gmultidcov(S,group,FALSE) # sill close to zero

Sigma = matrix(c(1,0.7,0,0.7,1,-0.2,0,-0.2,1), ncol=3)
X = MASS::mvrnorm(100, rep(0,3), Sigma)
gmultidcov(X,group,TRUE) # further from zero
gmultidcov(X,group,FALSE) # further from zero
```

---

infomaxICA

*Estimates independent components via infomax*

---

### Description

Estimate independent components using the infomax criteria, which is equivalent to maximum likelihood using the logistic density,  $\exp(-S)/(1+\exp(-S))^2$ .

### Usage

```
infomaxICA(X, n.comp, W.list = NULL, whiten = FALSE, maxit = 500, eps = 1e-08,
alpha.eps = 1e-08, verbose = FALSE, restarts=0)
```

**Arguments**

|                        |  |
|------------------------|--|
| <code>X</code>         | the $n \times p$ data matrix   |
| <code>n.comp</code>    | number of components to be estimated   |
| <code>W.list</code>    | list of orthogonal matrices for initialization   |
| <code>whiten</code>    | Whitens the data before applying ICA, i.e., $X\%*\%whitener = Z$ , where $Z$ has mean zero and empirical covariance equal to the identity matrix, and $Z$ is then used as the input.   |
| <code>maxit</code>     | maximum number of iterations   |
| <code>eps</code>       | algorithm terminates when the norm of the gradient is less than <code>eps</code>   |
| <code>alpha.eps</code> | tolerance controlling the level of annealing: algorithm terminates with a warning if the learning parameter is less than <code>alpha.eps</code>  |
| <code>verbose</code>   | if TRUE, prints (1) the value of the infomax objective function at each iteration, (2) the norm of the gradient, and (3) current value of the learning parameter <code>alpha</code> .  |
| <code>restarts</code>  | An integer determining the number of initial matrices to use in estimating the ICA model. The objective function has local optima, so multiple starting values are recommended. If <code>whiten=TRUE</code> , then generates random orthogonal matrices. If <code>whiten=FALSE</code> , generate random matrices from <code>mnorm()</code> . See code for details. |

**Details**

This is an R version of ICA using the infomax criteria that provides an alternative to Matlab code (<ftp://ftp.cnl.salk.edu/pub/tony/sep96.public>), but with a few modifications. First, we use the full data (the so-called offline algorithm) in each iteration rather than an online algorithm with batches. Secondly, we use an adaptive method to choose the step size (based upon Bernaards and Jennrich 2005), which speeds up convergence. We also omitted the bias term (intercept) included in the original formulation because we centered our data.

**Value**

|                          |   |
|--------------------------|---|
| <code>S</code>           | the estimated independent components  |
| <code>W</code>           | if <code>whiten=TRUE</code> , returns the orthogonal unmixing matrix; no value is returned when <code>whiten=FALSE</code>   |
| <code>M</code>           | Returns the estimated mixing matrix for the model $X = S M$ , where $X$ is not pre-whitened (although $X$ is centered)  |
| <code>f</code>           | the value of the objective function at the estimated $S$  |
| <code>Table</code>       | summarizes algorithm status at each iteration   |
| <code>convergence</code> | 1 if norm of the gradient is less than <code>eps</code> , 2 if the learning parameter was smaller than <code>alpha.eps</code> , which usually means the gradient is sufficiently small, 0 otherwise |

**Note**

In contrast to most other ICA methods, `W` is not constrained to be orthogonal.

**Author(s)**

Benjamin Risk

**References**

Bell, A. & Sejnowski, T. An information-maximization approach to blind separation and blind deconvolution *Neural computation*, 1995, 7, 1129-1159.

Bernaards, C. A. and Jennrich, R. I. (2005) Gradient Projection Algorithms and Software for Arbitrary Rotation Criteria in Factor Analysis, *Educational and Psychological Measurement* 65, 676-696. <<http://www.stat.ucla.edu/research/gpa>>

**Examples**

```
## Example when p > d. The MD function and amari measures
# are not defined for M. We can compare the
# "true W inverse", which is the mixing matrix multiplied
# by the whitening matrix; alternatively, we can use
# multidcov::frobICA. These two approaches are
# demonstrated below:

set.seed(999)
nObs <- 1024
nComp <- 3

# simulate from gamma distributions with
# varying amounts of skewness:
simS<-cbind(rgamma(nObs, shape = 1, scale = 2),
            rgamma(nObs, shape = 3, scale = 2),
            rgamma(nObs, shape = 9, scale = 0.5))

#standardize by expected value and variance:
simS[,1] = (simS[,1] - 1*2)/sqrt(1*2^2)
simS[,2] = (simS[,2] - 3*2)/sqrt(3*2^2)
simS[,3] = (simS[,3] - 9*0.5)/sqrt(9*0.5^2)

# slightly revised 'mixmat' function (from ProDenICA)
# for p>=d: uses fastICA and ProDenICA parameterization:
myMixmat <- function (p = 2, d = NULL) {
  if(is.null(d)) d = p
  a <- matrix(rnorm(d * p), d, p)
  sa <- La.svd(a)
  dL <- sort(runif(d) + 1)
  mat <- sa$u%*(sa$vt*dL)
  attr(mat, "condition") <- dL[d]/dL[1]
  mat
}

simM <- myMixmat(p = 6, d = nComp)
xData <- simS%*simM
xWhitened <- whitener(xData, n.comp = nComp)
#Define a 'true' W (uses the estimated whitening matrix):
```

```

W.true <- solve(simM%%xWhitened$whitener)

estInfomax <- infomaxICA(X = xData, n.comp = nComp, whiten = TRUE, verbose = TRUE)

frobICA(estInfomax$M, simM)
library(JADE)
MD(t(estInfomax$W), t(solve(W.true)))
amari.error(t(estInfomax$W), t(solve(W.true)))

```

---

matchICA

*match independent components using the Hungarian method*


---

### Description

The ICA model is only identifiable up to signed permutations of the ICs. This function finds the signed permutation of a matrix  $S$  such that  $\|S - \text{template}\|$  is minimized. Optionally also matches the mixing matrix  $M$ .

### Usage

```
matchICA(S, template, M = NULL)
```

### Arguments

|          |   |
|----------|---|
| $S$      | the $n \times d$ matrix of ICs to be matched  |
| template | the $n \times d$ matrix that $S$ is matched to.   |
| $M$      | an optional $d \times p$ mixing matrix corresponding to $S$ that will also be matched to the template |

### Value

Returns the signed permutation of  $S$  that is matched to the template. If the optional argument  $M$  is provided, returns a list with the permuted  $S$  and  $M$  matrices.

### Author(s)

Benjamin Risk

### References

Kuhn, H. The Hungarian Method for the assignment problem *Naval Research Logistics Quarterly*, 1955, 2, 83 - 97

Risk, B.B., D.S. Matteson, D. Ruppert, A. Eloyan, B.S. Caffo. In review, 2013. Evaluating ICA methods with an application to resting state fMRI.

### See Also

[frobICA clue::solve\\_LSAP](#)

**Examples**

```

set.seed(999)
nObs <- 1024
nComp <- 3
# simulate from some gamma distributions:
simS<-cbind(rgamma(nObs, shape = 1, scale = 2),
            rgamma(nObs, shape = 3, scale = 2),
            rgamma(nObs, shape = 9, scale = 0.5))

simM <- matrix(rnorm(9),3)
pMat <- matrix(c(0,-1,0,1,0,0,0,0,-1),3)
permS <- simS%%pMat
permM <- t(pMat)%%simM

matchedS <- matchICA(S = permS, template = simS, M = permM)
sum(abs(matchedS$S - simS))
sum(abs(simM - matchedS$M))

```

---

multidcov

*Symmetric multivariate distance covariance*


---

**Description**

Calculate either the symmetric or asymmetric multivariate distance covariance statistic.

**Usage**

```
multidcov(S, symmetric=TRUE, alpha=1)
```

**Arguments**

|                        |  |
|------------------------|--|
| <code>S</code>         | the $n \times d$ matrix for which you wish to calculate the dependence between $d$ columns from $n$ samples            |
| <code>alpha</code>     | A scaling parameter in the interval $(0,2]$ used for calculating distances.  |
| <code>symmetric</code> | logical; if TRUE (the default), calculates the symmetric version of the multivariate distance covariance. See details. |

**Details**

If `symmetric==TRUE`, calculates:  $\sum_i 1^d \text{dcovustat}(S[,i], S[,i])$  If `symmetric==FALSE`, calculates:  $\sum_i 1^{d-1} \text{dcovustat}(S[,i], S[(i+1):d])$

**Value**

returns a scalar equal to the multivariate distance covariance statistic for the columns of `S`

**Author(s)**

David Matteson

**See Also**

[dcovstat](#), [energy::dcov](#)

**Examples**

```
nObs <- 1024
nComp <- 3

simM <- matrix(rnorm(nComp*nComp),nComp)

# simulate some data:
simS<-cbind(rgamma(nObs, shape = 1, scale = 2),
            rgamma(nObs, shape = 3, scale = 2),
            rgamma(nObs, shape = 9, scale = 0.5))

simS <- scale(simS) #Standardize variance for identifiability

#mix the sources:
xData <- simS %*% simM

multidcov(simS) #close to zero
multidcov(whitener(xData)$Z) #should be larger than simS
multidcov(xData) #greater than zero
```

---

permTest

*Permutation test for mutual independence.*

---

**Description**

Calculates an approximate p-values based upon a permutation test for mutual independence.

**Usage**

```
permTest(S, group=1:ncol(S), R=199, FUN=c('gmultidcov','compInd'), ...)
```

**Arguments**

|       |  |
|-------|--|
| S     | The n x d matrix for which you wish to test the dependence between d columns from n samples      |
| group | A length d vector which indicates group membership for each component                            |
| R     | The number of permutations to perform in order to obtain the approximate p-value.                |
| FUN   | The function used to determine mutual independence. This is one of either gmultidcov or compInd. |
| ...   | Additional arguments passed to FUN. See details.   |

**Details**

Suppose that the groups are numbered 1,2,...,C and that group is a vector indicating group membership for each component. If symmetric==TRUE, calculates:  $\sum_{i=1}^C \text{dcovustat}(S[, \text{group}==i], S[, \text{group}!=i])$   
 If symmetric==FALSE, calculates:  $\sum_{i=1}^{C-1} \text{dcovustat}(S[, \text{group}==i], S[, \text{group}>i])$

If no additional arguments are supplied for FUN then the default values are used. In the case of gmultidcov, values for alpha and symmetric can be supplied. While for compInd only the value of alpha is needed.

**Value**

Returns an approximate p-values based upon a permutation test.

**Author(s)**

Nicholas James

**See Also**

[dcovustat](#), [energy::dcov](#)

---

rightskew

*force ICs to have positive skewness and order by skewness*

---

**Description**

The ICA model is only identifiable up to signed permutations. This function provides a canonical ordering for ICA that is useful for fMRI or studies where signals are skewed. Multiplies columns of S that are left-skewed by -1 to force right skewness. Optionally orders the columns by descending skewness.

**Usage**

```
rightskew(S, M = NULL, order.skew = TRUE)
```

**Arguments**

|            |  |
|------------|--|
| S          | n x d matrix   |
| M          | d x p mixing matrix  |
| order.skew | Option to return the permutation of columns of S from largest to smallest skewness. Also returns a permuted version of M that corresponds with the permuted S. |

**Value**

Returns the matrix S such that all columns have positive skewness. If optional argument M is supplied, returns a list with the new S and corresponding M.



**Author(s)**

Benjamin Risk

**References**

Eloyan, A. & Ghosh, S. A Semiparametric Approach to Source Separation using Independent Component Analysis Computational Statistics and Data Analysis, 2013, 58, 383 - 396.

**Examples**

```
nObs = 1024
simS<-cbind(rgamma(nObs, shape = 1, scale = 2),
            rgamma(nObs, shape = 9, scale = 0.5),
            -1*rgamma(nObs, shape = 3, scale = 2))

apply(simS,2,function(x){
  (sum((x - mean(x))^3)/length(x))/(sum((x - mean(x))^2)/length(x))^(3/2)})

canonicalS <- rightskew(simS)

apply(canonicalS,2,function(x){
  (sum((x - mean(x))^3)/length(x))/(sum((x - mean(x))^2)/length(x))^(3/2)})
```

steadyICA

*Estimate independent components by minimizing distance covariance***Description**

The model is:  $X = S M + E$ , where  $X$  is  $n \times p$  and has mean zero,  $S$  is  $n \times d$ ,  $M$  is  $d \times p$ , and  $E$  is measurement error. For whitened data, we have  $Z = S t(W)$ , where  $W$  is orthogonal. We find the matrix  $M$  such that  $S$  minimizes the distance covariance dependency measure.

**Usage**

```
steadyICA(X, n.comp = ncol(X), w.init = NULL, PIT = FALSE, bw = 'SJ', adjust = 1,
whiten = FALSE, irlba = FALSE, symmetric = FALSE, eps = 1e-08, alpha.eps = 1e-08,
maxit = 100, method = c('Cpp', 'R'), verbose = FALSE)
```

**Arguments**

|                     |   |
|---------------------|---|
| <code>X</code>      | The $n \times p$ data matrix, where $n$ is the number of observations.  |
| <code>n.comp</code> | number of components to be estimated  |
| <code>w.init</code> | a $p \times d$ initial unmixing matrix  |
| <code>PIT</code>    | logical; if TRUE, the distribution and density of the independent components are estimated using gaussian kernel density estimates. |
| <code>bw</code>     | Argument for bandwidth selection method; defaults to 'SJ'; see <code>stats::density</code>  |

|           |  |
|-----------|--|
| adjust    | adjust bandwidth selection; e.g., if observations are correlated, consider using <code>adjust &gt; 1</code> ; see <code>stats::density</code>  |
| whiten    | logical; if TRUE, whitens the data before applying ICA, i.e., $X\%*\%whitener = Z$ , where $Z$ has mean zero and empirical covariance equal to the identity matrix, and $Z$ is then used as the input. |
| irlba     | logical; when <code>whiten=TRUE</code> , <code>irlba=TRUE</code> uses the R-package 'irlba' in the whitening, which is generally faster than <code>base::svd</code> though sometimes less accurate     |
| symmetric | logical; if TRUE, implements the symmetric version of the ICA algorithm, which is invariant to the ordering of the columns of $X$ but is slower  |
| eps       | algorithm terminates when the norm of the gradient of <code>multidcov</code> is less than <code>eps</code>   |
| maxit     | maximum number of iterations   |
| alpha.eps | tolerance controlling the level of annealing: algorithm terminates with a warning if the learning parameter is less than <code>alpha.eps</code>  |
| method    | options 'Cpp' (default), which requires the package 'Rcpp', or 'R', which is solely written in R but is much slower  |
| verbose   | logical; if TRUE, prints the value of <code>multidcov</code> , norm of the gradient, and current value of the learning parameter.  |

**Value**

|             |   |
|-------------|---|
| S           | the estimated independent components  |
| W           | the estimated unmixing matrix: if <code>whiten=TRUE</code> , $W$ is orthogonal and corresponds to $Z W = S$ ; if <code>whiten=FALSE</code> , corresponds to $X \text{ginv}(M) = S$                  |
| M           | Returns the estimated mixing matrix for the model $X = S M$ , where $X$ is not pre-whitened (although $X$ is centered)  |
| f           | the value of the objective function at the estimated $S$  |
| Table       | summarizes algorithm status at each iteration   |
| convergence | 1 if norm of the gradient is less than <code>eps</code> , 2 if the learning parameter was smaller than <code>alpha.eps</code> , which usually means the gradient is sufficiently small, 0 otherwise |

**Author(s)**

Benjamin Risk

**References**

Matteson, D. S. & Tsay, R. Independent component analysis via U-Statistics. <<http://www.stat.cornell.edu/~matteson/#ICA>>

**See Also**

[multidcov](#)

**Examples**

```

set.seed(999)
nObs <- 1024
nComp <- 3
# simulate from some gamma distributions:
simS<-cbind(rgamma(nObs, shape = 1, scale = 2),
            rgamma(nObs, shape = 3, scale = 2),
            rgamma(nObs, shape = 9, scale = 0.5))

#standardize by expected value and variance:
simS[,1] = (simS[,1] - 1*2)/sqrt(1*2^2)
simS[,2] = (simS[,2] - 3*2)/sqrt(3*2^2)
simS[,3] = (simS[,3] - 9*0.5)/sqrt(9*0.5^2)

# slightly revised 'mixmat' function (from ProDenICA)
# for p>=d: uses fastICA and ProDenICA parameterization:
myMixmat <- function (p = 2, d = NULL) {
  if(is.null(d)) d = p
  a <- matrix(rnorm(d * p), d, p)
  sa <- La.svd(a)
  dL <- sort(runif(d) + 1)
  mat <- sa$u%*(sa$vt*dL)
  attr(mat, "condition") <- dL[d]/dL[1]
  mat
}

simM <- myMixmat(p = 6, d = nComp)
xData <- simS%*simM
xWhitened <- whitener(xData, n.comp = nComp)

#estimate mixing matrix:
est.steadyICA.v1 = steadyICA(X = xData,whiten=TRUE,n.comp=nComp,verbose = TRUE)

#Define the 'true' W:
W.true <- solve(simM%*xWhitened$whitener)

frobICA(M1=est.steadyICA.v1$M,M2=simM)
frobICA(S1=est.steadyICA.v1$S,S2=simS)

## Not run:
#now initiate from target:
est.steadyICA.v2 = steadyICA(X = xData, w.init= W.true, n.comp = nComp, whiten=TRUE, verbose=TRUE)

#estimate using PIT steadyICA such that dimension reduction is via ICA:
est.steadyICA.v3 = steadyICA(X = xData, w.init=ginv(est.steadyICA.v2$M),
PIT=TRUE, n.comp = nComp, whiten=FALSE, verbose=TRUE)

frobICA(M1=est.steadyICA.v2$M,M2=simM)
frobICA(M1=est.steadyICA.v3$M,M2=simM)
frobICA(S1=est.steadyICA.v2$S,S2=simS)

#tends to be lower than PCA-based (i.e., whitening) methods:

```

```
frobICA(S1=est.steadyICA.v3$S,S2=simS)

# JADE uses a different parameterization and different notation.
# Using our parameterization and notation, the arguments for
# JADE::amari.error correspond to:
amari.error(t(W.hat), W.true)

library(JADE)

amari.error(t(est.steadyICA.v1$W), W.true)
amari.error(t(est.steadyICA.v2$W), W.true)
##note that a square W is not estimated if PIT=TRUE and whiten=FALSE

#Compare performance to fastICA:
library(fastICA)
est.fastICA = fastICA(X = xData, n.comp = 3, tol=1e-07)
amari.error(t(est.fastICA$W), W.true)
##steadyICA usually outperforms fastICA

##Compare performance to ProDenICA:
library(ProDenICA)
est.ProDenICA = ProDenICA(x = xWhitened$Z, k = 3, maxit=40,trace=TRUE)
amari.error(t(est.ProDenICA$W), W.true)
##ProDenICA and steadyICA tend to be similar when sources
##are continuously differentiable

## End(Not run)
```

---

theta2W

---

*Convert angles to an orthogonal matrix.*


---

## Description

Convert  $d*(d-1)/2$  angles from a sequence of Givens rotations to a  $d \times d$  orthogonal matrix.

## Usage

```
theta2W(theta)
```

## Arguments

|       |   |
|-------|---|
| theta | A scalar or vector of length $d*(d-1)/2$ of values from which the $d \times d$ orthogonal matrix is calculated. |
|-------|---|

## Value

A  $d \times d$  orthogonal matrix resulting from the sequence of  $d*(d-1)/2$  Givens rotation matrices.

## Author(s)

David S. Matteson

**References**

Golub, G. & Van Loan, C. 1996. Matrix computations. Johns Hopkins University Press.

**See Also**

[W2theta](#)

**Examples**

```
#Generate orthogonal matrix:
mat <- matrix(rnorm(9),3,3)
W = svd(mat)$u

theta <- W2theta(W)

#Recovers W:
theta2W(theta)
```

---

W2theta

*Convert an orthogonal matrix to its angular parameterization.*

---

**Description**

Convert a  $d \times d$  orthogonal matrix to a sequence of  $d*(d-1)/2$  Givens rotations.

**Usage**

```
W2theta(W)
```

**Arguments**

W                    A  $d \times d$  orthogonal matrix.

**Details**

A  $d \times d$  orthogonal matrix can be decomposed into a series of  $d*(d-1)/2$  Givens rotation matrices, where each matrix is parameterized by a single angle.

**Value**

A vector of length  $d*(d-1)/2$  comprised of the angles.

**Author(s)**

David S. Matteson

**References**

Golub, G. & Van Loan, C. 1996. Matrix computations. Johns Hopkins University Press.

**See Also**[theta2W](#)**Examples**

```
theta = c(pi/6,pi/4,pi/2)

(W = theta2W(theta))

#Recover theta:
W2theta(W)
```

---

|          |                           |
|----------|---------------------------|
| whitener | <i>Whitening function</i> |
|----------|---------------------------|

---

**Description**

Subtract column means and transform columns such that the empirical covariance is equal to the identity matrix. Uses the SVD.

**Usage**

```
whitener(X, n.comp = ncol(X), center.row = FALSE, irlba = FALSE)
```

**Arguments**

|                         |   |
|-------------------------|---|
| <code>X</code>          | <code>n x p</code> matrix   |
| <code>n.comp</code>     | number of components to retain, i.e., first <code>n.comp</code> left eigenvectors from svd are retained       |
| <code>center.row</code> | center both rows and columns prior to applying SVD (the resulting whitened data does not have zero-mean rows) |
| <code>irlba</code>      | if TRUE, uses irlba to approximate the first <code>n.comp</code> left eigenvectors. See Note.                 |

**Value**

|                       |  |
|-----------------------|--|
| <code>whitener</code> | the matrix such that $X \%*\% \text{whitener}$ has zero mean and covariance equal to the identity matrix |
| <code>Z</code>        | the whitened data, i.e., $X \%*\% \text{whitener} = Z$   |

**Note**

The use of the option `'irlba = TRUE'` requires the package `irlba` and is very useful for large `p`. The function `irlba` only calculates the first `n.comp` eigenvectors and is much faster than `svd` for `p` » `n.comp`, for e.g., in `groupICA` of fMRI data.

**Author(s)**

Benjamin Risk

**See Also**

[svd](#), [irlba::irlba](#)

**Examples**

```
simData <- cbind(rnorm(1000,1,2),rnorm(1000,-1,3),rnorm(1000,4,1))
simMVN <- simData%%matrix(rnorm(12),3,4)
simWhiten <- whitener(simMVN,n.comp = 3)
colMeans(simWhiten$Z)
cov(simWhiten$Z)
```

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